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Expanded Optimization for Discovering Optimal Lateral Handling Bicycles

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Abstract

Physical design features of ground vehicles can affect their lateral handling qualities. Geometry, mass, and mass distribution of the vehicle's primary components as well as tire characteristics are primary contributors to poor and good handling due to their important influence on the vehicle's dynamics. In past work, we have presented a theoretical and computational framework for assessing the lateral task-independent handling qualities of simplified single track vehicle designs [1, 4]. In subsequent work, we showed that minimizing our handling quality metric (HQM) can produce theoretically optimal handling designs when only four geometric parameters are explored as the optimization variables [3]. The present work's goal is to expand this optimization problem to 24 geometry, mass, and inertial parameters as the optimization variables so that a broader search space of optimal bicycle designs is considered. This task is complicated by the fact that we desire to limit the search space to physically realizable designs. Doing so introduces 35 linear and nonlinear constraint equations on the optimization variables. We formulate a constrained optimization problem and use nonlinear programming to discover optimal, yet realizable, bicycle designs.

Our formulation exposes 24 optimization variables $\mathbf{p} \in \mathbb{R}^{24}$ that are both reformulations of and additions to the parameters of the benchmark bicycle [2]. We separate the rider's inertial parameters from the rear frame of the bicycle for the purposes of rider/bicycle relative configuration but do not introduce any more than the benchmark's four rigid bodies. The variables are as follows;

- wheelbase, trail, steer axis tilt, wheel radii
- somersault angle of the person (same direction as bicycle pitch)
- masses of the front frame, rear frame, front wheel, and rear wheel
- mass center locations of the front frame, rear frame, and rider
- principal central radii of gyration of front and rear frames
- principal inertia axis pitch angles of the front frame, rear frame, and rider

We formulate bounds on the variables as well as linear and nonlinear constraints among the variables to ensure a physically realizable bicycle. Below the constraints are defined and grouped by the associated rigid body or collection thereof:

Vehicle The resulting combination of the five rigid bodies.

- The physical extents of the bicycle and rider must exist above the ground plane.
- Both bicycle and rider are symmetric about the sagittal plane.
- The mass of each bicycle rigid body is positive, greater than a minimum value, and the total mass is below a reasonably lift-able amount.
- The wheels cannot overlap, i.e. wheelbase > front radius + rear radius.
- The bicycle cannot topple forward during hard braking or backward during hard acceleration, i.e. -1 g < acceleration < 1 g.

Wheels Both front and rear wheels have identical constraints.

- Wheel radius must be greater than a minimum value.
- Wheels are inertially wheel-like, i.e. $I_{rot} = mr^2, I_{rad} = I_{rot}/2$.

Frames Both front and rear frames have identical constraints.

- A pair of principal directions lie in the roll/yaw plane to maintain symmetry and one of these axes is within $\pm\pi/4$ rad of the ground plane.
- The mass and inertia of the frames are positive and large enough to be constructed from materials in a space frame of specified minimal density.
- The frames are planar in nature, i.e. $k^2_{lateral} = k^2_{longitudinal} + k^2_{vertical}$, where k is the radius of gyration.

Rider A single rigid body represents the rider.

- Rider mass is that of an average person.
- The rider's joint angles are fixed in a nominal configuration typical of upright bicycling and the resulting mass distribution is derived from standard body segment estimation methods.
- One rider principal axis is parallel with the bicycle pitch axis to enforce symmetry.

The constraint statements above translate into upper and lower bounds on 12 of the 24 variables $\mathbf{p}^U, \mathbf{p}^L$ and 14 linear and 21 nonlinear constraint $\mathbf{g}(\mathbf{p})$ among the variables and thus can be expressed in the standard non-linear programming form.

$$\begin{aligned} \underset{\mathbf{p}}{\text{minimize}} \quad & max(\text{HQM}(\mathbf{p})) \\ \text{subject to} \quad & \\ & \mathbf{p}^L \leq \mathbf{p} \leq \mathbf{p}^U \\ & \mathbf{g}^L \leq \mathbf{g}(\mathbf{p}) \leq \mathbf{g}^U \end{aligned} \tag{1}$$

We make use of the interior-point optimizer IPOPT [5] to find solutions to this nonlinear programming problem at a variety of vehicle speeds and will present the resulting discovered optimal bicycle designs in the full conference paper.

References

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