

Lagrange's Equations

Lagrangian

$$L = K - V$$

↑ ↑
total total
Kinetic potential
energy energy
of system of a system

If only conservative forces are present, Lagrange showed that Newton-Euler equations of a system defined by n generalized coordinates q_1, \dots, q_n can be found with:

$$\underbrace{\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_r} \right)}_{\text{map to } F_r^*} - \underbrace{\frac{\partial L}{\partial q_r}}_{\text{maps to } F_r \text{ (only the conservative portions)}} = 0 \quad \text{for } r=1, \dots, n$$

Lagrange's equation of the second kind.

Constraints

$$\dot{\bar{q}} = \bar{u}$$

$$f_{nh}(\dot{\bar{q}}, \bar{q}, t) = 0$$

$$\rightarrow M_{hn} \ddot{\bar{q}} + g_{hn}(\bar{q}, t) = 0 \quad \leftarrow \text{motion constraint}$$

$$M_d \ddot{\bar{q}} + g_d(\dot{\bar{q}}, \bar{q}, t) = 0 \quad \leftarrow \text{dynamical differential equations}$$

$$\begin{bmatrix} M_d & M_{hn}^T \\ M_{hn} & 0 \end{bmatrix} \begin{bmatrix} \ddot{\bar{q}} \\ \lambda \end{bmatrix} = \begin{bmatrix} -\bar{g}_d \\ -\bar{g}_\lambda \end{bmatrix}$$

M_a M_a g_a

$$(1) M_d \ddot{q} + \underbrace{M_{hn}^T \bar{\lambda}} + \bar{g}_d = 0$$

$$(2) \underbrace{M_{hn} \ddot{q} + \bar{g}_2}_{\text{time derivative of the motion constraints}} = 0$$

time derivative
of the motion
constraints

constraint
generalized
forces

$\bar{\lambda}$ Lagrange multipliers \rightarrow the measure numbers
of the constraint forces and torques
(non contributing)